On the Accuracy Improvement of Low-Power Orientation Filters Using IMU and MARG Sensor Arrays

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Abstract— The orientation estimation filter proposed by Madgwick [1] for inertial and inertial/magnetic sensors have been successfully used as a core solution in a variety of commercial low-power motion tracking devices. The approach offers a high accuracy and reduces the computational cost compared to the state-of-the-art Kalman-based methods [10]. In this paper, we propose a modification to the Gradient Descent correction step in [1], which guarantees the improvement of accuracy in quaternion estimation without increasing the total number of required arithmetic operations. Monte Carlo simulations with typical statistical characteristics for sensor errors have shown an average 37.2% improvement in dynamic root-mean-square error of Euler angles.

I. INTRODUCTION

Orientation estimation of a moving object in 3-dimensional space using Inertial Measurement Unit (IMU) and Magnetic Angular Rate and Gravity (MARG) sensor arrays has vast applications in gaming, avionics, wearable devices, etc. The orientation estimation can be performed either online using a microcontroller on the device, or offline using a separate computer. While offline methods can handle complex calculations, for real-time purposes, the system is mostly required to be low-cost and low-power, which justifies the utilization of low-cost microcontrollers. Consequently, targeting particularly low-power real-time applications and motion tracking wearable devices, the orientation estimation method should not involve intensive arithmetic computations.

Calibration [3-4], filtering, and fusion algorithms are typically implemented as part of the system firmware to perform orientation estimation with a high enough accuracy, while satisfying some real-time constraints. In general, state of the art orientation filters provide a trade-off between computational cost (applicability to real-time systems), use of additional sensors [5-6], and accuracy of estimation.

Conventional orientation models using IMU and MARG sensors are based on either using direct Euler angle representations, or the quaternion representation. Quaternion representation is computationally efficient and unlike Euler angle matrices, does not suffer from singularity conditions.

Several solutions exist in the literature, which make use of quaternions to represent orientation. Namely, Kalman-based methods [8-9] have been extensively used for this purpose. Although being accurate, such methods are not suitable for low-power embedded applications, as they involve matrix inversions and covariance matrix calculations. Namely, the extended Kalman filter proposed in [9] is shown to be about 20 times slower compared to the non-Kalman-based orientation filters in [1] and [7], when implemented on the powerful Cortex M4 microcontroller, while offering an average improvement of only 9% in the dynamic Root-Mean-Square (RMS) error of orientation angles.

Madgwick’s orientation filter presented in [1] is a computationally-efficient method, which makes use of the quaternion representation and is suitable for low-power real-time applications. The approach delivers a high enough accuracy and avoids the time-consuming calculations required by the Kalman-based methods. It uses the gradient descent algorithm. Madgwick’s orientation filter has been used as a core building-block unit in several commercial low-power motion-detection and motion-tracking devices [10-11].

Several adaptations can be applied to the approach in [1] to achieve better accuracy. For instance, the method in [14], introduces efficient ways to determine the presence of disturbance in sensor readings and remove temporary glitches. This can help to adaptively set the Madgwick’s filter gain [1] to tolerate temporary disturbance in sensor readings. The filter can even be implemented on a low-cost and low-power microcontroller, such as ARM Cortex M0, which supports only integer and fixed-point [12] arithmetic operations.

This paper introduces an Orientation Model for Inertial Devices (OMID), which is essentially an improved version of the Madgwick filter in [1]. All variations applicable to the core solution in [1] can be applied to OMID as well. The proposed filter guarantees the improvement of the overall filter accuracy without increasing the number of required arithmetic operations. An extended gradient descent solution is proposed that uses the information from not only accelerometer and magnetometer readings at time $t$, but also gyroscopes at time $t$. The gradient descent step at time $t$ presented in [1] does not use any information from gyro readings at time $t$. This limits the efficiency of the overall filter, which is resolved here.

Monte Carlo simulations over different sensor error characteristics and reference angular velocities have shown an average 37.2% improvement in the root-mean-square error of
Euler angles compared to the Madgwick filter. The improvement of the proposed filter is magnified, when gyro show lower precision, or when the device moves faster.

The rest of this paper is organized as follows. Section II addresses the definitions and notations. Section III presents the proposed filter, OMID and its advantages over Madgwick’s solution [1]. Section IV presents the Monte Carlo simulation results. Finally, Section V concludes the paper.

II. DEFINITIONS AND NOTATIONS

The following definitions will be used in this paper.

Definition 1: We denote the Euler angles in the reference (Earth) frame with North, East and Up vectors. We define the Euler angles in the so-called aerospace sequence, where rotation around z-axis (Yaw) takes place first, which is then followed by Pitch (around y-axis), and Roll (around x-axis).

Definition 2: The IMU/MARG sensor frame (ridged body) uses $\tilde{x}$, $\tilde{y}$, $\tilde{z}$ as the principal axes for sensor readings.

Definition 3: The unit-length quaternion

$$q_{r,t} = [q_{r,t,1} q_{r,t,2} q_{r,t,3} q_{r,t,4}], \quad (\|q_{r,t}\| = 1),$$

where $\|\|$ is the norm, represents the actual orientation of the sensor frame relative to the Earth frame at time $t$. The conjugate of $q_{r,t}$, defined below, will swap the relative frames:

$$q_{r,t}^* = [q_{r,t,1} - q_{r,t,2} - q_{r,t,3} - q_{r,t,4}].$$

Definition 4: We denote the quaternion estimate at time $t$ found by the orientation filter as $\hat{q}_{est,t}$.

Definition 5: We denote the current calibrated accelerometer, magnetometer, and gyroscope readings in the sensor frame at time $t$ as $S_{a,t} = (a_x, a_y, a_z)$, $S_{m,t} = (m_x, m_y, m_z)$, and $S_{g,t} = (g_x, g_y, g_z)$, respectively. The current accelerometer and magnetometer (Acc/Mag) readings at time $t$ are together represented as $S_{am,t} = \{S_{a,t}, S_{m,t}\}$. Note that regarding the IMU filter, we have: $S_{am,t} = S_{a,t}$.

III. THE PROPOSED ORIENTATION FILTER

In this section we present the proposed filter targeting IMU and MARG sensor arrays. The overall computations follow the high-level block diagrams shown in Fig. 1 and Fig. 2 regarding the IMU and MARG sensor arrays, respectively. Here, we describe the functionality of each block.

Initial Calibration and Filtering: The IMU and MARG sensor readings are assumed to have initially passed the necessary calibration, filtering, and normalization techniques. For instance, low-pass/high-pass filters might be used to remove the undesirable frequencies. Calibration techniques aim to remove the offset and gain from sensor readings [3-4].

Gyro Quaternion Corrector: This block directly predicts the quaternion derivative $\dot{q}_{g,t}$ using the information from gyros $S_{g,t}$ and the previous estimate $q_{est,t-1}$ as follows [1]:

$$\dot{q}_{g,t} = \frac{1}{2} q_{est,t-1} \otimes [g_x, g_y, g_z]. \quad (1)$$

where $\otimes$ is the Hamilton product.

Figure 1. Proposed IMU filter block diagram.

Integrator: This block finds $(A)/(B)$ in Fig. (1)/Fig. (2):

$$q_{g,t} = q_{est,t-1} + \hat{q}_{g,t} \Delta t. \quad (2)$$

where $\Delta t$ is the sampling period and $\hat{q}_{g,t}$ is given by Eq. (1).

Acc/Mag Quaternion Corrector: This block predicts a correction vector $\hat{q}_{vt}$, i.e., Node (C) in Fig. 2, that pushes its initial guess $q_{init} = q_{g,t}$ towards a unit-length quaternion $q_{am,t}$ that ideally matches the Acc/Mag readings $S_{am,t}$, i.e.,

$$\hat{q}_{vt} = q_{init} - q_{am,t} = q_{g,t} - q_{am,t}. \quad (3)$$

In case of the IMU filter, the block is called Accelerometer Quaternion Corrector and it generates Node (B) in Fig. 1. We will use a single-step gradient descent solution with the initial guess $q_{vt}$ given by Eq. (2), to find $\hat{q}_{vt}$ as follows:

$$\hat{q}_{vt} = \nabla F(q_{g,t}, S_{am,t}), \quad (4)$$

where $\nabla F$ is the gradient of $F$, which is given below:

$$F(q_{g,t}, S_{am,t}) = \int \frac{1}{2} F_g(q_{g,t}, S_{am,t})^T F_g(q_{g,t}, S_{am,t}).$$

The function $F_g$ is arbitrarily chosen to represent the mismatch between $S_{am,t}$ and $q_{g,t} = [q_1, q_2, q_3, q_4]$. For the IMU filter, we can use the following mismatch function:

$$F_g(q_{g,t}, S_{a,t}) = q_{g,t} \otimes g \otimes q_{g,t} - S_{a,t} \quad (5)$$

where $g = [0\:0\:0\:1]$ is the gravity vector in the Earth frame. Note that if we replace $q_{g,t}$ with $q_{est,t-1}$ in Eq. (5), i.e., change the initial guess $q_{init}$ for the gradient descent, the calculation of $\nabla F$ becomes identical to the approach in [1]. Hence, the number of required arithmetic operations remains the same.

Figure 2. Proposed MARG filter block diagram.
Regarding MARG sensors, one might define the mismatch function $F_g$ as a $6 \times 1$ matrix [1]. We might also adaptively scale $F_g$ w.r.t. the presence of disturbance in $S_{a,t}$ and $S_{m,t}$ [14].

The gradient in Eq. (4) is then computed as follows:

$$q_{\vec{t}t} = \nabla F(q_{\vec{g}t}, S_{\text{am},t}) = J_{F_g}(q_{\vec{g}t})F_g(q_{\vec{g}t}, S_{\text{am},t}), \tag{6}$$

where $J_{F_g}(q_{\vec{g}t})$ is the Jacobian of $F_g$ at $q = q_{\vec{g}t}$. Considering $F_g$ in Eq. (5) for the IMU filter, we get:

$$J_{F_g}(q_{\vec{g}t}) = \begin{bmatrix} -2q_3 & 2q_4 & -2q_1 & 2q_2 \\ 2q_2 & 2q_3 & 2q_4 & 2q_1 \\ 0 & -4q_2 & -4q_3 & 0 \end{bmatrix}_{3 \times 4}. \tag{7}$$

**Final Adder:** The adder finds Node (C)/(E) in Fig. (1)/(2):

$$\hat{q}_{\text{est},t} = q_{\vec{g}t} - (\beta \Delta t) \hat{q}_{\vec{t}t}, \tag{8}$$

which is followed by a normalizer at the end. The tunable gain $\beta$ can be set adaptively based on sensor characteristics and the presence of disturbance in sensor readings to achieve optimal accuracy just like [1]. We might also set a default optimal $\beta$, where initially a higher gain is chosen for faster convergence.

**Gyro Zero-Bias Drift Estimator:** The zero-bias drift effect of gyro can be compensated in both IMU and MARG cases by using the mean value of the gyro readings in stationary positions [14]. However, such methods are not capable of removing the bias drift, while being in motion. For that purpose, we can alternatively use this block for the MARG filter to predict the zero-bias drift $S_{b,t}$, i.e., (D) in Fig. (2).

The zero-bias value for gyro can be represented by the DC component of the gyros’ quaternion error at time $t$, i.e., $(2q_{\text{est},t-1} \otimes \hat{q}_{\vec{t}t})$. It can be found using an integrator:

$$S_{b,t} = \xi \sum_t (2q_{\text{est},t-1} \otimes \hat{q}_{\vec{t}t}) \Delta t, \tag{9}$$

where $\xi$ is another filter gain representing the response time in tracking the zero-bias drift. The bias values $S_{b,t}$ are initially subtracted from the gyro readings $S_{\vec{g}t}$ and later, the corrected readings $S_{\vec{g}t} - S_{b,t}$, i.e., Node (A) in Fig. 2, are fed through the gyro quaternion corrector block, which delivers $q_{\vec{g}t}$.

Once again, if we replace $q_{\vec{g}t}$ in Eq. (6) with $q_{\text{est},t-1}$, the calculation of $\nabla F$ becomes identical to the Madgwick’s core solution [1]. However, our method majorly improves the overall accuracy. This claim has been analytically addressed in details in [15] due to the lack of space here. Monte Carlo simulations using the Scilab tool are presented next.

### IV. SIMULATION RESULTS

In this section we evaluate the efficiency of OMID compared to [1]. Monte Carlo simulations over different sensor characteristics and reference angular velocities have been performed to provide a detailed comparison. To make a fair comparison, a sweep over the gain $\beta$ has been performed on each filter separately to find its optimal value, which minimizes the Euler angles’ RMS error. This means that each filter benefits from its own optimal gain considering the given sensor error characteristics and the reference motion pattern.

Accelerometer and magnetometer errors are modeled with a zero-mean Gaussian noise with the standard deviation of 0.03, which corresponds to a typical error around 3 degrees. For gyros, we have used two zero-mean Gaussian error scenarios, one with the standard deviation of 1°/s (Scenario#1 with little noise), and another one with the standard deviation of 2°/s (Scenario#2 with more noise).

In the first experiment we have generated 10,000 samples in Scilab with the sampling frequency of 100 Hz to represent the reference motion pattern. The pattern follows rotations in pitch, as partially depicted by Fig. 3. The angular velocity starts high and it slows down until pitch reaches 160 degrees. Next, the rotation direction changes until we reach 0 degrees in pitch at a faster pace. The rotation changes direction again and starts to slow down. The reference motion has been generated with the following characteristics:

Average absolute angular velocity in pitch: 71.4°/s.
Maximum absolute angular velocity in pitch: 114.6°/s.

![Figure 3. The reference pitch angles showing variable angular velocities.](image)

The results from a) gyros only ($\beta = 0$), b) Madgwick filter [1], and c) the proposed method, OMID are shown in Table I. The proposed filter delivers the superior accuracy and its improvement is magnified as gyros show lower precision.

In the second experiment we compare the proposed MARG filter and its counterpart in [1] under relatively faster motions with the following characteristics:

Average absolute angular velocity in pitch/yaw: 87.32°/s.
Maximum angular velocity in pitch/yaw: 262.62°/s.
Average absolute angular velocity in roll: 82.17°/s.
Maximum absolute angular velocity in roll: 156.69°/s.

Table II summarizes the results. Compared to Table I, the improvements are magnified due to the quicker motions.

<table>
<thead>
<tr>
<th>TABLE I. COMPARISON OF THE IMU FILTERS IN TWO SCENARIOS</th>
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<tbody>
<tr>
<td>Estimation Method</td>
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<td>-------------------</td>
</tr>
<tr>
<td>Gyros only</td>
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<tr>
<td>Madgwick [1]</td>
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<tr>
<td>OMID</td>
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**28% improvement**

<table>
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<tr>
<th>TABLE II. COMPARISON OF MARG FILTERS UNDER QUICK MOTIONS</th>
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<tr>
<td>Estimation Method</td>
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<tr>
<td>-------------------</td>
</tr>
<tr>
<td>Gyros only</td>
</tr>
<tr>
<td>Madgwick ($\beta_{\text{opt}} = 0.023$)</td>
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**48.5% improvement**
In the final experiment we evaluate the efficiency of OMID and Madgwick filters in tracking gyro's zero-bias drift in motion. We make use of the same reference motion pattern in the first experiment with the gyro error models from Scenario#2. The optimal values of $\beta$ for Scenario#2 in Table II have been chosen for the filters. The bias-correction gain $\xi$ is set for the filters separately, such that it satisfies a 45-second step-response time to reach 90% of the steady-state value.

We have injected a reference time-varying offset to the gyro readings in pitch as shown in Fig. 4. Our filter outperforms the filter in [1] by delivering a much smoother response in tracking the drift.

Major improvements have also been observed in the final Euler angle calculation in the presence of zero-bias drift. Table III shows a major 79.8% improvement in dynamic RMS error in calculating the pitch angles. This improvement is based on the fact that gyros on their own are less accurate in the presence of zero-bias drift, which makes the proposed filter much more accurate compared to [1], as it allows for all sensor readings to affect the gradient descent correction.

![Tracking Gyro Pitch Offset](image)

**TABLE III. COMPARISON OF THE FILTERS IN PRESENCE OF DRIFT**

<table>
<thead>
<tr>
<th>Estimation Method</th>
<th>$\beta_{opt}$</th>
<th>$\xi$</th>
<th>Dynamic RMS error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Madgwick [1]</td>
<td>0.012</td>
<td>0.002</td>
<td>2.0628</td>
</tr>
<tr>
<td>OMID</td>
<td>0.32</td>
<td>0.0495</td>
<td>0.4171</td>
</tr>
</tbody>
</table>

V. CONCLUSION AND FUTURE WORK

This paper proposed an orientation filter, OMID, for IMU and MARG sensor arrays, which is an improved version of the filter proposed by Madgwick at its core level [1]. The filter delivers a higher accuracy without requiring additional arithmetic operations. The gradient descent correction step in [1] is modified to involve information from all sensor readings. Monte Carlo simulations have shown an average improvement of 37.2% in dynamic RMS error of Euler angles.

The Madgwick filter has been used as the core algorithm for more advanced orientation filters as well as many state-of-the-art low-power motion tracking devices. Consequently, the improved version of the filter, i.e., OMID, can be helpful in a variety of applications. We have implemented an extended adaptive version of the proposed filter on our Nebлина tracking module, which is about the size of a fingertip and it features a low-power microcontroller, a Bluetooth Low-Energy module, and a 9-axis MARG sensor array [13]. Fig. 5 shows a snapshot of our experiment on the Nebлина board to visualize the device’s self-calculated 3D orientation.

![OMID filter’s MARG implementation on Nebлина.](image)

REFERENCES


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2 A provisional patent has been filed on June 2015 to protect this invention.